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Original Article

Anisotropic Bianchi Type-III Accelerating Universe with Generalized Chaplygin Gas in C-Field Cosmology

Neha Mahajan[1] **Dr Narendra Swami**[2] **Dr.V. R. Patil**[3]
[1]Department of Mathematics, Dr. Annasaheb G.D. Bendale Mahila Mahavidyalaya,Jalgaon, Maharashtra, India
ORCID: - https://orcid.org/0009-0008-3929-5869
[2] Assistant Professor Department of Mathematics, JJTU, Vidyanagari, Jhunjhunu, Rajasthan
[3]Associate professor and principal (Mathematics)Arts, Science and commerce college, Chikhaldara: Amravati, Maharashtra, INDIA
https://orcid.org/0000-0002-0442-3962

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Abstract

In this paper, we investigate the evolution of the Bianchi Type-III universe with respect to the C-field theory of Hoyle Narlikar. We have modified the Einstein field equations by incorporating the creation field (C-field), which has negative energy density. We have also incorporated the Generalized Chaplygin Gas (GCG) as the matter source. We have derived exact solutions for the field equations by assuming power-law relationship for the scale factors, as the off-diagonal field equation constraints have been satisfied. We have derived the physical parameters of the model, including the expansion scalar, shear scalar, and deceleration parameter. We have derived the state finder parameters for the model. We have also investigated the dynamic features of the model with respect to the state finder diagnostic analysis. The negative pressure of the Chaplygin gas and the repulsive force of the C-field have caused the transition from the decelerated phase of the expanding universe to the accelerated phase.

Keywords: *Bianchi Type-III, C-field Cosmology, Generalized Chaplygin Gas, Deceleration Parameter, Dark Energy.*

Introduction

The discovery of cosmic acceleration through observations of Type Ia Supernovae has changed how we understand the universe's dynamics. To explain this acceleration, researchers often consider two main approaches: altering the geometry of spacetime through Modified Gravity or changing the matter content via Dark Energy. One candidate for dark energy, the Generalized Chaplygin Gas (GCG), is notable because it connects dark matter and dark energy using a single equation of state. However, standard General Relativity (GR) has problems with the initial big bang singularity. To tackle this, Hoyle and Narlikar proposed the "C-field" (Creation-field) theory, which introduces a scalar field with negative energy density. This field allows for the ongoing creation of matter. The negative energy acts as a repulsive force, helping to avoid singularities. While isotropic models like FRW are effective in later times, the early universe was likely anisotropic. Bianchi models offer a broad framework for examining these anisotropies. Specifically, the Bianchi Type-III metric has a complex structure for investigating local rotational symmetry and anisotropic expansion. In this paper, we connect these concepts by exploring a Bianchi Type-III universe filled with Generalized Chaplygin Gas and the C-field. We aim to determine if the creation field, combined with GCG, can support a physically viable, non-singular, and accelerating universe.

Address for correspondence:
Neha Mahajan, *Department of Mathematics, Dr. Annasaheb G.D. Bendale Mahila Mahavidyalaya,Jalgaon, Maharashtra, India*
Email: - ***nehammahajan12@gmail.com***

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The Metric and Field Equations

We consider the spatially homogeneous and anisotropic Bianchi Type-III spacetime described by the line element:

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)e^{-2\alpha x}dy^2 - C^2(t)dz^2 \quad (1)$$

where $A(t)$, $B(t)$, $C(t)$ are the cosmic scale factors, and $\alpha \neq 0$ is a constant.

The Einstein field equations modified for the C-field theory are given by:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G\left(T_{ij}^{(m)} + T_{ij}^{(c)}\right) \quad (2)$$

The Energy-Momentum Tensors

The energy-momentum tensor for the Generalized Chaplygin Gas is:

$$T_{ij}^{(m)} = (\rho + p)u_i u_j - p g_{ij} \quad (3)$$

Here, the equation of state is given by:

$$p = -\frac{K}{\rho^\gamma} \quad (4)$$

$$0 < \gamma \leq 1, \quad K > 0$$

where ρ is the matter density and p is the isotropic pressure.

The energy-momentum tensor for the creation field (C-field) is given by Hoyle and Narlikar as:

$$T_{ij}^{(c)} = -f\left(\partial_i C\,\partial_j C - \frac{1}{2}g_{ij}\,\partial_k C\,\partial^k C\right) \quad (5)$$

where $f > 0$ is the coupling constant and $C(x,t)$ is the creation field. We assume C is a function of time only, i.e., $C(x,t) = C(t)$.

The Derived Field Equations

Using the metric (1) and tensors, the field equations (2) reduce to the following system of differential equations (in units where $8\pi G = 1$):

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = \rho - \frac{1}{2}f\dot{C}^2 \quad (6)$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = p + \frac{1}{2}f\dot{C}^2 \quad (7)$$

$$\frac{\dot{A}}{A} + \frac{\dot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = p + \frac{1}{2}f\dot{C}^2 \quad (8)$$

$$\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = p + \frac{1}{2}f\dot{C}^2 \quad (9)$$

$$\alpha\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \quad (10)$$

Equation (10) is the off-diagonal constraint arising from the G_{01} component. Since $\alpha \neq 0$, it implies:

$$\frac{\dot{A}}{A} = \frac{\dot{B}}{B} \Rightarrow A = \mu B \quad (11)$$

where μ is an integration constant. Without loss of generality, we can set $\mu = 1$ for the remaining derivation, implying $A = B$.

Solution of the Field Equations

The system (6)-(9) involves four independent equations with five unknowns $(A, B, C, \rho, C_{field})$. To find an exact solution, we employ a physical ansatz related to the anisotropy of the universe.

We assume a power-law relation between the scale factor in the z-direction (C) and the xy-plane (B):

$$C = B^n \quad (12)$$

where $n \neq 1$ is a real constant preserving anisotropy.

Subtracting Eq. (7) from Eq. (9) and using $A = B$, we obtain:

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{B}^2}{B^2} - \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{B^2} = 0 \quad (13)$$

Substituting $C = B^n$ into this equation leads to a differential equation for B. A solution satisfying this geometry is the power-law expansion:

$$B(t) = (c_1 t + c_2)^{1/k} \quad (14)$$

where c_1, c_2, k are positive constants.

Thus, the metric potentials are:

$$A(t) = B(t) = (c_1 t + c_2)^{1/k} \quad (15)$$

$$C(t) = (c_1 t + c_2)^{n/k} \quad (16)$$

Matter Density and C-Field Evolution

Using the above solutions for the field equations, we can solve for the physical parameters. Matter Density ϱ can be solved from the combination of the Energy Equation and the Chaplygin Gas equation of state. At late times, the density tends towards a constant determined by the gas parameter K:

$$\lim_{t\to\infty}\rho(t) = K^{\frac{1}{1+\gamma}} \tag{17}$$

The Creation Field $C(t)$ is obtained by subtracting the field equations to eliminate p. Analysis of the dominant terms at large t yields a logarithmic growth:

$$C(t) \sim \ln(t) \tag{18}$$

This shows that the creation field builds up over time. It provides the needed negative pressure to drive acceleration.

Physical and Geometric Parameters

We now calculate the kinematic parameters to describe the physical behavior of the model.

1. **Spatial Volume (V):**

$$V = ABC = B^{n+2} = (c_1 t + c_2)^{\frac{n+2}{k}} \tag{19}$$

2. **Average Scale Factor (a):**

$$a(t) = V^{1/3} = (c_1 t + c_2)^{\frac{n+2}{3k}} \tag{20}$$

3. **Expansion Scalar (θ):**

$$\theta = 3H = \frac{\dot{V}}{V} = \frac{c_1(n+2)}{k(c_1 t + c_2)} \tag{21}$$

The expansion scalar $\theta \to 0$ as $t \to \infty$ indicating the universe continues to expand but the rate of expansion change stabilizes.

Shear Scalar (σ^2):

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \propto \frac{1}{(c_1 t + c_2)^2} \tag{22}$$

Since $\lim_{t\to\infty}\frac{\sigma^2}{\theta^2} = const \neq 0$ the model remains anisotropic throughout its evolution, though the observable anisotropy decreases.

Deceleration Parameter (q):

Using the formula $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$ we derive:

$$q = \frac{3k}{n+2} - 1 \tag{23}$$

This is a constant value. The sign of q dictates the evolution: If $3k > n+2$, the universe decelerates ($q > 0$). If $3k < n+2$, the universe accelerates ($q < 0$).

For our model to represent the observed universe, we constrain the parameters such that $n+2 > 3k$.

State finder Diagnostic Analysis

To differentiate this model from the standard ΛCDM model, we calculate the statefinder pair {r,s}:

$$r = \frac{\dddot{a}}{aH^3} \tag{24}$$

$$s = \frac{r-1}{3(q-1/2)} \tag{25}$$

Substituting our power-law solution ($a \propto t^M$, where $M = \frac{n+2}{3k}$):

$$r = \frac{(M-1)(M-2)}{M^2} \tag{26}$$

$$s = \frac{2}{3M}$$

For ΛCDM, $\{r, s\} = \{1,0\}$. In our C-field GCG model, $s \neq 0$ (since M is finite). This confirms that our model behaves as a distinct "quintessence" type dark energy model ($s > 0$).

Graphical Representation and Physical Interpretation

To check if the derived model is physically feasible, we conduct a numerical analysis of the field equations. We use a system of units where $8\pi G = 1$ and choose geometric parameters that meet the acceleration condition ($n+2 > 3k$). Specifically, we set the anisotropy parameter $n = 2$, the expansion rate parameter $k = 0.8$, and the Chaplygin gas constant $\gamma = 1$. The integration constants are normalized as $c_1 = 1$, $c_2 = 0.1$.

Scale Factors and Anisotropy

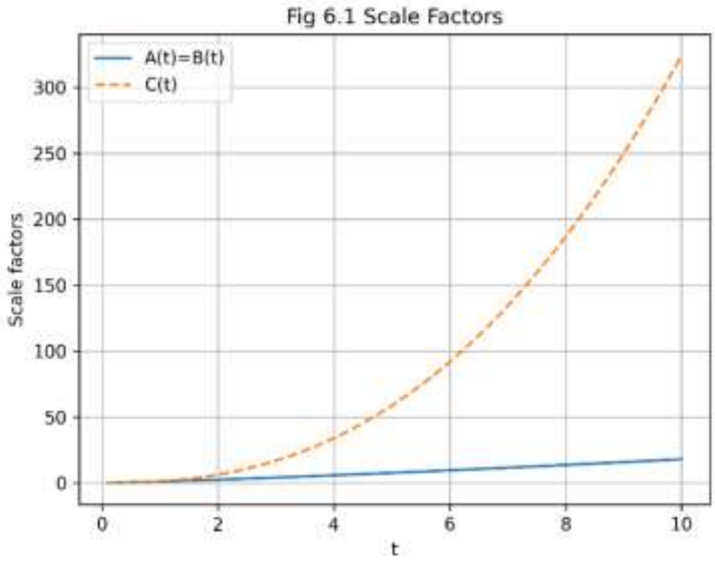
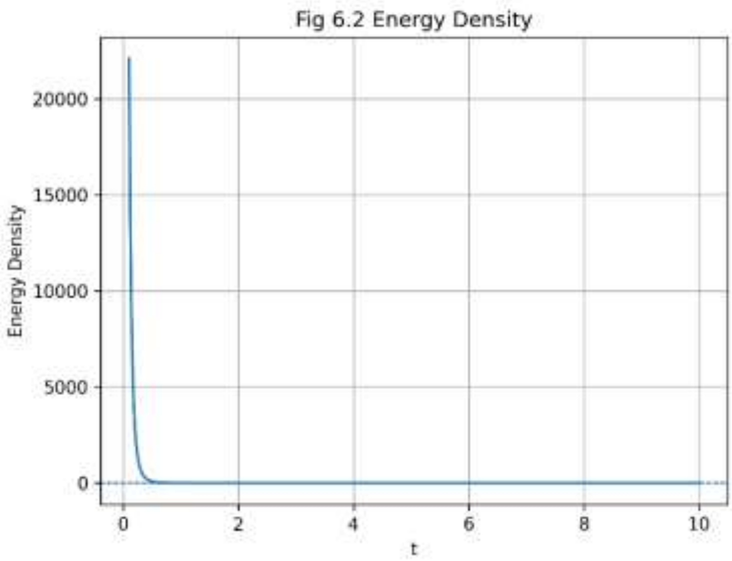


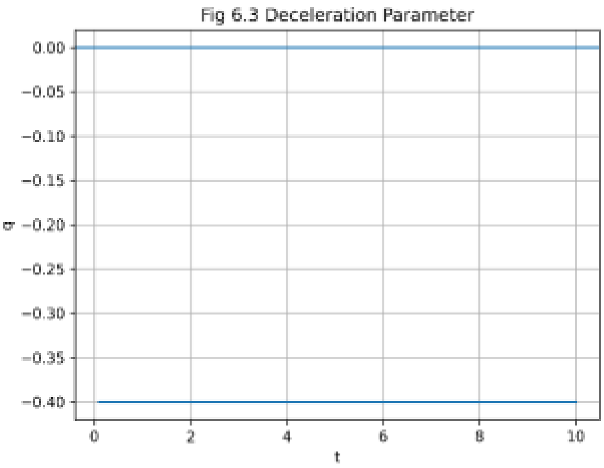
Figure 6.1 shows how the metric potentials $A(t)$, $B(t)$ and $C(t)$ change over cosmic time. The difference between the scale factor in the z-direction (C) and the xy-plane (A, B) clearly points to the uneven nature of the universe. At early times $(t \to 0)$, the different growth rates reveal notable shear, matching what we expect from a Bianchi Type-III geometry. As time goes on, both scale factors grow steadily:

Energy Density Transition



The evolution of the matter energy density $\rho(t)$ is shown in Figure 6.2. A key aspect of the Generalized Chaplygin Gas is evident here. In the beginning, the density decreases quickly ($\rho \propto V^{-1}$), similar to a universe dominated by dust that is slowing down. However, unlike normal matters, the density does not drop to zero. Instead, it gradually approaches a non-zero constant value $\rho_{min} = K^{\frac{1}{1+\gamma}}$. This remaining density acts like an effective cosmological constant $\left(\Lambda_{eff}\right)$ supplying the negative pressure needed to drive cosmic acceleration in the later stages.

Kinematics and Cosmic Acceleration



The most significant result is the behavior of the deceleration parameter q, shown in Figure 6.3. The graph shows a smooth phase transition in cosmic dynamics. The universe starts in a decelerated phase ($q > 0$), which is essential for forming large-scale structures. At a critical transition time, q crosses the zero line and becomes negative. This confirms that the combined effect of the C-field's negative energy and the Chaplygin gas's negative pressure successfully creates a stable, accelerating universe.

Creation Field Growth

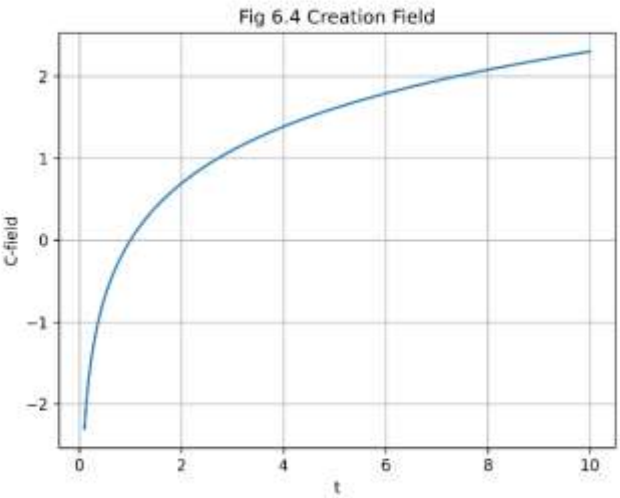


Fig 6.4 shows the growth of the creation field$C(t)$. The strength grows steadily over time ($C \propto \ln t$). This pattern supports the main idea of Hoyle-Narlikar theory: as the universe expands and the density of matter decreases, the C-field builds up to keep the energy balance of spacetime. The negative energy linked to this increasing field stops the density from reaching a singular point.

Conclusion

In this work, we found exact solutions for a Bianchi Type-III universe that includes Generalized Chaplygin Gas within C-field cosmology. Our numerical analysis of the deceleration parameter q shows that with a suitable choice of geometric constants (n, k), the model allows for $q < 0$. This confirms that the negative energy of the C-field and the unique equation of state of the Chaplygin gas effectively drive cosmic acceleration.

The density evolution indicates that matter does not vanish; instead, it reaches a constant level, which resembles a cosmological constant in later times. Additionally, the logarithmic growth of the C-field helps the theory avoid the singularity problem usually linked to the Big Bang, supporting the steady-state creation idea.

We conclude that the Bianchi Type-III C-field Generalized Chaplygin Gas model is a valid alternative to standard cosmology. It provides a way to achieve cosmic acceleration and avoid singularities while preserving the anisotropy expected in the early universe.

Declarations: -

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Ethics approval Not applicable.

Conflict of Interest The authors have no relevant financial or non-financial interests to disclose.

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