

## Original Article

# Applications of Derivatives: Conceptual Understanding of Maxima and Minima Using Derivative Tests

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Manuscript ID:  
BN-2026-030224

ISSN: 3065-7865

Volume 3

Issue 2

February 2026

Pp. 126-129

Submitted: 15 Jan 2026

Revised: 27 Jan 2026

Accepted: 15 Feb 2026

Published: 28 Feb 2026

DOI:

[10.5281/zenodo.19850508](https://doi.org/10.5281/zenodo.19850508)

DOI link:

<https://doi.org/10.5281/zenodo.19850508>



Quick Response Code:



Website: <https://bnir.us>



### Abstract

The concept of maxima and minima represents one of the most fundamental applications of differential calculus, with far-reaching implications across mathematics, physics, engineering, economics, and numerous other disciplines. The theoretical underpinnings and real-world uses of derivative tests for locating and categorizing function critical points are examined in this essay. We look at the First Derivative Test, Second Derivative Test, and Higher-Order Derivative Tests, offering a thorough framework for comprehending how derivatives show how functions behave at extremal locations.

Through rigorous mathematical exposition and practical examples, this study demonstrates how derivative tests serve as powerful analytical tools for optimization problems. Through dynamic and interactive graph visualization, the Desmos Graphing Calculator improves mathematics comprehension. It facilitates pupils' understanding of abstract ideas through lucid visual aids.

**Keywords:** Derivatives, Maxima, Minima, Critical Points, First Derivative Test, Second Derivative Test, Optimization, Calculus and Desmos Graphing Calculator.

### Introduction

#### Historical Context and Motivation

The development of differential calculus in the 17th century by Isaac Newton and Gottfried Wilhelm Leibniz revolutionized mathematical analysis and provided systematic methods for solving optimization problems that had challenged mathematicians for centuries. The concept of using derivatives to locate maxima and minima emerged as one of the most powerful applications of this new mathematical framework, enabling precise solutions to problems ranging from determining the most efficient geometric shapes to optimizing resource allocation in complex systems.

The fundamental question addressed by extremum theory is deceptively simple: given a function  $f(x)$ , how can we systematically identify points where the function attains its largest or smallest values? This question underlies countless practical problems: engineers seeking to minimize material costs while maximizing structural strength, economists analyzing profit maximization under resource constraints, physicists determining equilibrium states of physical systems, and biologists modeling population dynamics.

#### Significance and Scope

Understanding maxima and minima through derivative tests represents a cornerstone of mathematical education and professional practice. The derivative, defined as the instantaneous rate of change of a function, provides direct insight into the function's behavior. At points where a function transitions from increasing to decreasing (or vice versa), the derivative must equal zero or fail to exist, creating what we term "critical points." The systematic analysis of these critical points through derivative tests enables us to classify them as local maxima, local minima, or saddle points.

This paper presents a comprehensive examination of derivative tests for extrema, structured as follows: Section 2 establishes the theoretical foundations and formal definitions; The First Derivative Test and its geometric meaning are covered in Section 3;

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#### How to cite this article:

Thomare, L. Y. (2026). Applications of Derivatives: Conceptual Understanding of Maxima and Minima Using Derivative Tests. *Bulletin of Nexus Journal*, 3(2), 126–129.

<https://doi.org/10.5281/zenodo.19850508>

the Second Derivative Test and its connection to function concavity are covered in Section 4. Section 5 discusses higher-order tests and special cases; Section 6 presents applications across multiple disciplines; Section 7 integration of technology ; and Section 8 concludes with future directions.

### Fundamental Definitions

(1) A **critical point** (where a local maximum or minimum occurs) is defined as a point ( $x=c$ ) within the domain of  $f$ , where either  $f'(c)=0$  or  $f'(c)$  does not exist.

(2) A **local maximum** occurs at  $c$  if there is an interval  $(a,b)$  around  $c$ , such that the value of  $f(c)$  is greater than or equal to the value of  $f(x)$ , for any value of  $x$  that is located in the interval  $(a,b)$ .

(3) A **local minimum** occurs at  $x=c$  if there is an interval  $(a,b)$  around  $c$  and the value of  $f(c)$  is less than or equal to the value of  $f(x)$ , for any value of  $x$  that is located in the interval  $(a,b)$ .

**Global Extrema Description:** A point  $c$  of a real-valued function  $f$  is the greatest global (or absolute) point in the range of values of  $f$  if  $f(c)$  is greater than or equal to all values in the range of  $f$ . Furthermore, if  $f(c)$  is less than or equal to all values in the range, then  $c$  is the lowest global value of  $f$ .

### 3) FIRST DERIVATIVE TESTS

**THEORY & DEFINITION:** The first derivative test categorizes critical points using the point of inflection of  $f'(x)$ .  $f$  is continuous at a critical point  $c$  per Theorem 3.1 ("First Derivative Test").

$a) c$  is a point of maximum if  $f'(x)$  decreases through the zero of  $f'(c)$  (i.e., around  $c$ ,  $f'(x) > 0$  if  $x < c$  and  $f'(x) < 0$  if  $x > c$ ).

$(b)$ : If  $f$  has a local Minimum at  $c$ , this means that  $f'$  has changed from being Negative (for  $x < c$ ) to being Positive (for  $x > c$ ).

$(c)$ : If there is No change in Sign at  $c$  then it would be considered to Have No Local Maximum or Minimum, Only an Inflection point with a Horizontal Tangent Line.

### Geometric Interpretation

The relationship between the derivative of a function and its monotonicity provide a very direct geometric interpretation of the first derivative test. If  $f'(x)$  is greater than zero ( $f'(x) > 0$ ), then the function is increasing. If  $f'(x)$  is less than zero ( $f'(x) < 0$ ), then the function is decreasing. Therefore, when a function goes from increasing to decreasing, we have a local maximum; conversely, a local minimum occurs when the function goes from decreasing to increasing.

Examine the function  $f(x) = x^3 - 3x^2 - 9x + 5$ .

The derivative can be calculated as follows:  $f'(x) = 3x^2 - 6x - 9 = 3(x - 3)(x + 1)$ .

$x = -1$  and  $x = 3$  are the key locations.

For  $x < -1$ :  $f'(x) > 0$  (function growing); for  $x < 3$ :  $f'(x) < 0$  (function reducing).

$f'(x) > 0$  (function rising) for  $x > 3$ .

According to the First Derivative Test,  $f$  has a local minimum at  $x = 3$  and a local maximum at  $x = -1$ .

### Graphical Representation :

The graph of  $f(x) = x^3 - 3x^2 - 9x + 5$  was plotted using the Desmos Graphing Calculator. The behavior of the function is shown graphically in the interactive link that follows.

<https://www.desmos.com/calculator/y7iqngbr1u>

### Advantages and Disadvantages

When is the First Derivative Test particularly beneficial? • The second derivative is not available or computer-generated in your example. • The derivative of the function is not continuous and you need to evaluate the function. • You would like to know what the function is doing on the global scale. However, for multiple critical points, it is often a lot of work to look at the sign of  $f'$  on both sides.

### The Second Derivative Test

**Connection Between Concavity and the Statement** In addition to determining the classifications assigned to critical points, the Second Derivative Test can also be used to find out whether a function is concave/convex in the immediate area of the critical point by examining the value of the second derivative at that critical point. If we have  $f''(x)$  continuous in a neighborhood of  $c$  and  $f'(c) = 0$ , then according to

### Theorem 4.1 (the Second Derivative Test),

we have:  $(a)$   $f$  has a local minimum at  $c$  provided that  $f''(c) > 0$ .

$(b)$   $f$  has a local maximum at  $c$  provided that  $f''(c) < 0$ .

$(c)$  If  $f''(c) = 0$  then the Second Derivative Test yields inconclusive results. If  $f''(c) > 0$ , this means  $f$  is concave upwards in the neighborhood of  $c$  and  $f$  will have a local minimum (i.e., it turns upward like a cup). Conversely, if  $f''(c) < 0$ , this means  $f$  is concave downwards in the neighborhood of  $c$  and will therefore have a local maximum (i.e., it turns downwards like a bowl).

**Real Life Example:**  $f(x) = x^3 - 3x^2 + 2$  We can find the Critical points by finding When  $f'(x) = 0 \Rightarrow f'(x) = 3x^2 - 6x = 3x(x-2)$  (when  $f'$  changes signs) ( $x=0$  and  $x=2$  are critical points). Let's find  $f''(x)$ : for  $f'(x) = 6x - 6$  Let's Apply the Second Derivative Test to find the nature of the Critical points at 0 and 2.  $f''(0) = 6(0) - 6 = -6$  (This tells us that  $x=0$  has a Local Maximum)  $f''(2) = 6(2) - 6 = 6$  (This means that  $x=2$  has a local minimum)

### Visual Display:

The graph of  $f(x) = x^3 - 3x^2 + 2$  was plotted using the Desmos Graphing Calculator. The behavior of the function is shown graphically in the interactive link that follows:

<https://www.desmos.com/calculator/nphzmy2cjd>

**When the Test Is Unsuccessful :**

When  $f''(c) = 0$ , the Second Derivative Test yields no decisive results. Traditional instances consist of:

- Although  $x = 0$  is a local minimum,  $f(x) = x^4$  has  $f'(0) = f''(0) = 0$ .
- (<https://www.desmos.com/calculator/yqehxxtppzz>)
- Although  $x = 0$  is a local maximum,  $f(x) = -x^4$  has  $f'(0) = f''(0) = 0$ .
- (<https://www.desmos.com/calculator/0cyryuo5ew>)
- Although  $x = 0$  is an inflection point,  $f(x) = x^3$  has  $f'(0) = f''(0) = 0$ .

(<https://www.desmos.com/calculator/ozmk8w88gg>)

The First Derivative Test or higher-order derivative tests must be used in these situations.

**Tests of Higher Order Derivatives****General (nth) Derivative Test**

If both  $f'(c)=0$  and  $f''(c)=0$ , the usage of the higher order derivative may sometimes determine the classification. **Theorem 5.1 (nth Derivative Test)** If  $f^{(n)}$  is continuous in the vicinity of  $c$ ,  $f'(c)=f''(c)=\dots=f^{(n-1)}(c)=0$ , but  $f^{(n)}(c)\neq 0$ , then:

- If  $n$  is even and  $f^{(n)}(c)>0$ ,  $f$  has a local minimum at  $c$ .
- If  $n$  is even and  $f^{(n)}(c)<0$ ,  $f$  has a local maximum at  $c$ .
- If  $n$  is odd,  $f$  does not have a local maximum or minimum at  $c$  (inflection point).

This test provides a systematic method for difficult situations and generalizes the Second Derivative Test.

**Application Example**

Consider  $f(x) = x^5 - 5x^3$ . Computing derivatives:  $f'(x) = 5x^4 - 15x^2 = 5x^2(x^2 - 3)$

$f''(x) = 20x^3 - 30x = 10x(2x^2 - 3)$

At  $x = 0$ :  $f'(0) = 0$  and  $f''(0) = 0$  (inconclusive)

Computing higher derivatives:  $f'''(x) = 60x^2 - 30$   $f'''(0) = -30 \neq 0$

Since the first non-zero derivative is the third (odd order) and is negative,  $x = 0$  is an inflection point, not an extremum.

**Graphical Representation :**

The Desmos Graphing Calculator was used to plot the graph of  $f(x) = x^5 - 5x^3$ .

The following interactive link provides a graphic representation of the function's behavior:

<https://www.desmos.com/calculator/xletsz3pi4>

**Applications Across Disciplines****Engineering and Design Optimization**

In engineering, derivative tests are essential for optimizing designs subject to constraints. Consider the problem of designing a cylindrical can with

volume  $V = 1000 \text{ cm}^3$  that minimizes surface area (and thus material cost).

Let  $r$  be the radius and  $h$  the height. The constraint gives  $h = 1000/(\pi r^2)$ , and the surface area is:  $A(r) = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2000/r$

Finding the critical point:  $A'(r) = 4\pi r - 2000/r^2 = 0$   
 $4\pi r^3 = 2000$   $r = (500/\pi)^{1/3} \approx 5.42 \text{ cm}$

Applying the Second Derivative Test:  $A''(r) = 4\pi + 4000/r^3$   $A''(5.42) > 0$  (local minimum)

This confirms the optimal radius minimizes material usage.

**Machine Learning and Data Science**

In modern machine learning, optimization algorithms like gradient descent rely on derivatives to minimize loss functions. The derivative indicates the direction of steepest descent, guiding iterative updates to model parameters. While these applications often involve multivariable calculus, the fundamental principle of using derivatives to find minima remains central.

**Integration of Technology: Desmos Graphing Calculator**

The following are some ways that the Desmos Graphing Calculator improves studying mathematics:

The ability to graph algebraic, trigonometric, exponential, logarithmic, parametric, and implicit functions beyond manual calculation is provided by the Exploration of Complex Functions.

- Instant Visual Feedback: Using graphical representation, this tool aids students in comprehending concavity, derivatives, critical points, and function behavior.
- Dynamic "What-If" Analysis: Slider features allow you to change parameters in real time and see changes like reflections, stretches, and shifts.
- Finding Maxima and Minima: This method uses graphical analysis and derivative visualization to help find the local and global maximum and minimum points of functions.
- Numerical and Graphical Verification: This feature allows you to verify analytical solutions using numerical estimation and graphical intersections.
- Inquiry-based and interactive learning promotes experimentation and a deeper comprehension of concepts.

**Conclusion**

The examination of maxima and minima using derivative tests, which offer powerful tools for analyzing function behavior and addressing optimization problems, is one of calculus's fundamental achievements. The First Derivative Test connects monotonicity to the derivative's sign, while the Second Derivative Test connects extrema to concavity and, when suitable, provides a useful

method. Together, these studies demonstrate the relationship between the local linear approximation and the overall function behavior.

Derivative tests represent fundamental mathematical concepts including continuity, differentiability, and logical analysis in addition to computation. Their significance extends to variational calculus, optimization theory, and contemporary computing applications. To understand why these techniques are effective, one must expand their conceptual knowledge beyond procedural stages. Derivative-based methods for examining extremal behavior continue to be crucial in both theoretical and applied contexts as mathematics applications spread across a wide range of domains.

#### Acknowledgment

I would like to express my sincere gratitude to everyone who supported and guided me during the completion of this project on **“Applications of Derivatives: Conceptual Understanding of Maxima and Minima Using Derivative Tests.”**

First and foremost, I extend my heartfelt thanks to my mathematics teacher for providing valuable guidance, continuous encouragement, and clear explanations that helped me understand the concepts of derivatives and their applications more effectively. Their support played a vital role in developing my conceptual clarity and analytical thinking.

I am also thankful to my school for providing the necessary resources and a positive learning environment that made this work possible. My appreciation goes to my classmates and friends for their helpful discussions, cooperation, and motivation throughout the project.

I would like to express my gratitude to my parents and family members for their constant encouragement, patience, and moral support during the preparation of this assignment.

#### Financial support and sponsorship

Nil.

#### Conflicts of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper

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