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Original Article

# Mathematical Approaches to Climate Change Prediction

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***Abstract***

*Climate change has emerged as one of the most critical global challenges, requiring reliable prediction and scientific understanding to support policy and decision-making. Mathematics plays a fundamental role in climate science by providing theoretical frameworks and computational tools to analyze complex climate systems. This study examines the mathematical approaches used in climate change prediction, focusing on dynamical systems, stochastic processes, statistical inference, and numerical modeling techniques. Mathematical models help describe interactions among atmospheric, oceanic, and terrestrial components while capturing nonlinear dynamics and long-term variability. Methods such as finite difference and finite element techniques, grid-based models, and data assimilation strategies—including Kalman filtering—enhance the accuracy of climate simulations and forecasts. The paper also highlights the role of general circulation models, energy balance models, and multi-scale modeling frameworks in projecting future climate scenarios. Furthermore, it discusses uncertainty quantification and probabilistic forecasting methods that improve the reliability of climate predictions. By integrating mathematical theory with computational modeling and statistical analysis, these approaches contribute significantly to understanding climate dynamics and supporting sustainable environmental planning.*

***Keywords:*** *Climate Change Prediction; Mathematical Modeling; Dynamical Systems; Stochastic Processes; Numerical Methods; General Circulation Models; Data Assimilation; Uncertainty Quantification.*

## Introduction

Mathematics underlies climate science. Mathematical models specify the climate system, elucidate physical mechanisms, and link observations to polar climate's dynamics. Fundamental issues remain despite extensive applied mathematics on climate change. Mathematics communicates basic principles of climate and climate change since the Renaissance.

Mathematics describes aspects of climate systems through dynamical systems, stochastic systems, statistics, numerical analysis, and decision making (Brayshaw, 2018). These interconnections foster understanding of model formation, parameterization, uncertainty, and risks posed by climate change (Thompson and Sieber, 2013). Climate—averaged over extended periods—recedes into a mix of weather and climate mathematical understanding, yet climate systems exhibit complex dynamics that challenge human familiarity with nonlinear behaviors. Despite substantial modeling progress, predictions do not consistently meet societal needs (Lucarini et al., 2015).

## Mathematical Foundations for Climate Modeling

Mathematics plays a crucial role in climate science, straddling the gap between abstract theories and data-driven decision making. Dynamical systems, statistics, and stochastic processes are common modeling abstractions in atmospheric, oceanic, and climate science. Deterministic physical laws lead to nonlinear partial differential equations (PDEs) governing the large-scale dynamics of the atmosphere and oceans. Global equilibrium states are sought to analyze details of the chaotic attractor and their long-term influence on mean-flow and zonal-mean models. Statistical models express correlations among climate variables and enable estimates of future condition probabilities conditioned upon the present values.



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These models identify relationships among observations instead of describing the physical processes that shape them and fit into the statistical paradigm of climate modeling. Recent years have witnessed a surge in interest in nonlinear stochastic processes as a mechanism explaining climate variability. Stochastic processes found in climate modeling literature include Brownian motion, Lévy processes, and Markov chains, which correspond to finite-dimensional nonlinear deterministic systems. Stochastic differential equations (SDEs) are introduced to describe the nonlinear influence of additive and multiplicative white-in-time noise across a variety of processes. Noise models vary in complexity and include seasonal, interannual, and multidecadal components depending on temporal scale. The effects of forcing and Laplacian dissipation are usually included to align with realistic climate dynamics, despite certain nonlinear processes having negligible impact.

Modern climate models divide the Earth into spatial boxes approximated as a physical system with few degrees of freedom. The balance among different terms captures specific phenomena of interest at the selected time-scale. Typically, separate models are constructed for various components of the Earth System (atmosphere, oceans, land surface, vegetation) and formally incorporated into the climate model. Such models can also be converted into statistical-data-driven emulators offering a greater understanding of the slow manifold associated with climate systems. The time evolution of a climate model generates trajectories in an extremely high-dimensional phase space. In the climate projection scenario, greenhouse-gas concentrations are configured according to a chosen pathway. The focal point of parameter estimation comprises the identification of physically and scientifically relevant quantities bankrolling the projections of a climate model (Thompson and Sieber, 2013). A clear distinction is drawn between the projection scenarios and the auxiliary control. The latter allows the synchronization of different climate models to the same observational record and remains constant throughout the simulation. The climate model encompasses many components, each characterized by, usually, a vector of parameters and has a full set of initial conditions designated the value of the state vector at time (Benney and Radulescu, 2015).

**Dynamical Systems and Differential Equations**

A climate system can be considered as a deterministic dynamical system whose state varies over a finite-dimensional manifold characterized by a specific number (degrees of freedom) of temperature and flow-like pattern components. Such parameters determine the different sets of local and nonlocal physical processes activated by the energy input coming from solar radiation and volcanism that interact with the corresponding spatial variables. The degree of freedom can be extended by incorporating the snow and ice cover to allow the model to handle climate changes and their associated long-term effects. An example of this model is given by a dynamical system defined separately on the continents. Such climate models exhibit several interesting dynamical features, comparable to those observed in the real climate system, including multi-stability of different climatic states, existence of multiple-periodic orbits, glacial-interglacial cycles, stochastic transitions between stable attractors, and bifurcation phenomena.

Finite-dimensional deterministic dynamical models can also be derived from infinite-dimensional ocean-atmosphere geophysical fluid mathematical models that provide valuable insight into the long-term behavior of the climate system. In addition to governing energy and momentum conservation equations, both dynamical systems and stochastic differential equations can be established together with nonlinear dissipation mechanisms. The different time periods of variability in the external forcing (30-year solar cycle and 106-year geological period) interacting with an internal time scale of about 600 years observed in classical zonally averaged atmosphere-ocean general circulation climate models operate on a significantly slower time scale (Dunmyre et al., 2019).

**Stochastic Processes in Climate Variability**

The climate of the Earth undergoes natural variations on a wide range of time and space scales (Berner et al., 2017). Climate science recognizes the presence of both deterministic and stochastic components in models of the evolution of the climate system. Randomness, arising from physical or biological sources, governs variability at time scales too short for a meaningful description of the true underlying system. Another source of randomness comes from uncertainties in model parameterization of unresolved processes and related structural uncertainties. Climate models form a crucial part of climate science. The future state of the Earth's climate determines the character of heat and moisture exchanges between land, ocean, and atmosphere, and hence the future of ecosystems and biodiversity, of food security and irrigation systems, of health security from vector-borne diseases, and many other factors affecting the future of humanity

Given the complexity of climate systems, stochastic climate-models—both conceptual and composed of stochastic differential equations (SDEs)—are widely used to study the effects of noise on the mean climate and also on extreme-value statistics. Climate models often have inherent noise and stochasticity at sub-grid levels of description. Randomness contributes to the deposition of water vapour in the atmosphere through condensation nuclei and aerosol-cloud-interaction mechanisms. It is also at the heart of many critical climate processes. For example, inclement weather, precipitable water, and vapour transport for the future electrolyte process are influenced by chaotic behaviour of zonal winds thanks to ocean-atmosphere coupling in El Niño/Southern-Oscillation (ENSO). Low frequency modes are also linked to climatic variables, such as sea surface temperature and ice coverage, which in turn influence the occurrence of collective fishery collapse. Surface pressure anomalies in the North and South poles govern up to 51% of spatial-temporal variability in precipitation in the Tibetan Plateau Precipitation Climate Events. Stochastic parameterization schemes enhance the representation of unresolved physical processes related to stochastic weather types and also coupled ocean-wave-atmospheric extremes.

Stochastic fluctuations play a vital role in the determination of climate-statistics and extreme-event statistics. Stochastic–partial-differential-equation (SPDE) methodologies in terms of the primitive equations are developed. Large-scale and unresolved processes that control weather and hence climate are explicitly modelled.

**Statistical Inference and Parameter Estimation**

The complexities and non-linearities of the climate system significantly complicate the task of parameter estimation. Given the dynamic, multi-scale, and multi-process nature of climate models, both permanent and evolving climate parameters are often indistinct. Standard strategies to carry out the parameter estimation problem focus on either prior specification and selection or optimum parameter identification. Climate models being intrinsically built on physical principles, strong guidance for prior choice can thus be derived directly from these fundamentals. Because temperature, precipitation, sea level, and other climate characteristics develop over decades to centuries marine bio geochemical models can generally be viewed as fast dynamical systems. Time-averaged statistics can consequently be used for either prior characterisation or credible interval definition. Additionally, the simulation of multi-decadal climate change requires the systematic solution of the climate parameter estimation problem. Perturbed observations disseminated through consecutive decades from a drifting re-analysis constitute a practical framework, subject to both parametric and structural uncertainty.

The scientific community devoted to climate modelling has recently acknowledged the range of uncertainties arising during climate prediction and the importance of appropriate characterisation of uncertainty within probabilistic climate projections. Wherever essential, parameters which specify climate model components such as the atmosphere, ocean, land-surface, or cryosphere and which are introduced to describe uncertain processes (e.g equations governing the exchange between land-ice and ocean, or permafrost and ground temperature, etc.) become the central focus of study. Occurring at nearly every spatial location and dominant in many climate-model human-induced emissions may have an existing or prevailing dependence upon species or concentration levels already forecast (García-Pintado and Paul, 2018).

Numerical Methods for Climate Prediction

Mathematical modeling provides a coherent framework to describe and analyze climate processes. Dynamical systems, widely used in climate models, have been pivotal in understanding the mechanisms governing the climate system and its behavior under varied external forcing. Ensemble simulations and other techniques support identifying underlying physical processes and improve quantification of future climate change projections. Identification of the most relevant noise processes for the climate system reveals complex structures closely linked to the underlying dynamics. Such insights enhance understanding of climate variability and change and facilitate more efficient forecasting. Use of stochastic models and complementing simulations with short, stochastic prototypes improve predictive capabilities and help assess confidence in projections of future climate. Automation of such procedures enables consideration of a broader range of models and model forms.

Numerical methods play a critical part in transferring mathematical climate models into usable predictive tools. First, the mathematical form must be established and the spectrum of physical processes that are relevant anatomized. Since the governing equations of these mathematical models typically possess infinite dimension and feature a highly non-linear character, an adequate approximation must be pursued. This entails a discretization of the spatial domain and the construction of a numerical scheme to propagate the discretized equations in time. Appropriate selection of discretization methods arises from consideration of the climate phenomena to be simulated and the governing conservation principles. Truncation error analysis of the numerical scheme illuminates the class of climate scenarios from which useful projections can be obtained. Numerical stability analysis discloses time-steps that economize computation without jeopardizing the reliability of predicted behaviour. Heat-wave formation in specific regions remains projected by such models despite a robust combination of parameter perturbation and model-dimensionality reduction, signalling their ongoing utility.

**Discretization Techniques and Grid-Based Models**

Grid-based models and discretization techniques constitute an essential sub-field of climate modeling. Grid-based models simulate climate dynamics over a fixed set of spatial locations corresponding to a structured spatial grid. The climate state is represented by a set of state variables that are defined at each grid location. Discretization methods describe how to approximate the governing partial differential equations of climate dynamics at these grid locations. For each time step, physical processes are computed at the grid points—typically using local information and spatial interpolations—then integrated in time using a discretization scheme. Computer resources typically limit the maximum size of grids, thus constraining the degree of detail that can be realistically addressed. Typical grid sizes range from 100 km or larger for global models to between 20 and 100 km for regional models (R. Sain et al., 2011).

The simulation of spatial dynamical models often employs a consistent approach to time evolution over spatial discretization. The evolution equations for roughly 80% of climate models, including all major general circulation models, are based on partial differential equations that may be discretized as advection-convection-diffusion equations. The representation of these equations follows closely standard procedures used in the numerical simulation of polluted-air-atmosphere transport, ocean circulation, and many other processes of interest (J. Challinor et al., 2013).

**Finite Difference and Finite Element Methods**

Finite Difference and Finite Element Methods

Discretization methods reduce climate dynamics equations to large systems of algebraic equations defined on the spatial–temporal domain. Climate models typically adopt grid-based discretizations of both space and time, where the model domain is partitioned into discrete subdomains (grids) and the evolution of the climate system state is computed at discrete time steps. Coupled climate models decouple the governing equations associated with each climate process and iterate among these equations during the time-stepping scheme.

Finite difference (FD) and finite element (FE) methods differ on the discretization approach: FD methods approximate the model equations at specific grid points in the domain, while FE methods construct global approximate solutions from local low-order approximation spaces. For linear discretized climate model equations, FE methods can be expressed in a weak formulation, which allows generic variational principles to apply to these models. Although nonlinear climate model equations have also been tackled with precise strong formulations on unstructured grids, reasonable analysis of climate model approximation errors for both linear and nonlinear discretizations is generally facilitated by low-order FD models.

FD methods can be classified at the spatial level by the type of average used to approximate spatial derivatives within each grid box. Other key distinctions concern how time-stepping schemes are coupled with spatial discretization. High-order explicit multi-step time schemes, which support long time steps without resorting to overly restrictive spatial discretization, are compatible with explicit low-order triangular unstructured models. Coupling water transport via shallow-water model equations, these formulations can simulate significant surface flow propagation and scaling accurately along a damped sine-wave wavelet.

FD methods yield instantaneous information about spatial derivatives but limit temporal information to the most recent time step, leading to significant numerical instabilities when solving hyperbolic systems via a wave-front-velocity concept. Including temporal discretization via the implicit second-order Crank–Nicolson scheme, zero-lag boundary conditions extend numerical simulation capability. The explicit formulation with variable time and depth increments nevertheless permits wider applicability and gains in flexibility. FE methods are generally more complex to implement than FD methods but yield similar or increased global accuracy (Berger et al., 2019) and naturally accommodate unstructured grids well-balanced with spatially variable physical model properties, avoiding zeroth-order schemes.

Global FE discretizations of the linear advection–diffusion–reaction equation on general meshes exhibit algebraic convergence rates in output error matching the optimal Lagrangian interpolation of moving fronts. When considering space–time dimension spaces of general finite dimension, coupled space–time FE discretizations can be constructed that preserve these convergence rates in combinations of $L^\infty(L^2)$ and $L^2(H^1)$ norms, short-wave reflection in Frequency Fourier error analysis relates to the choice of unconditionally stable schemes as an extension to semilinear problems.

**Data Assimilation and Kalman Filtering**

Data assimilation provides a statistically optimal estimate of a dynamical system's current state by combining model predictions with observations. Originally developed for weather forecasting, the Kalman filter and its variants have since been applied in fields such as geoscience, petroleum engineering, and medicine. The estimates produced support, among other tasks, parameter estimation and future state prediction. Recently, data assimilation schemes have been proposed to calibrate complex climate models (R. Hunt et al., 2005).

The goal of climate forecasting is to predict future states of a climate model based on the model itself and currently available information. Such predictions rely on estimates of model parameters and initial conditions. Climate data assimilation addresses the problem of estimating these components. By making optimal use of model predictions and observations, data assimilation schemes improve the estimates of parameter values and the initial state of the climate system. When the model and the climate system share similar time scales, data assimilation schemes can also serve to predict future climate states, thereby enhancing forecast skill.

Predictive Frameworks for Climate Projections

Numerous climate models aim to capture climate dynamics and project future conditions (K. Niven, 2011). A critical challenge lies in the wide range of time and space scales involved. Atmospheric circulation models, for instance, can exhibit accurate simulations at a daily time step with relatively small domains. Such models remain of limited value for long-term climate predictions because the atmospheric component alone would fail to encompass sufficiently large spatial and temporal scales. In this circumstance, general circulation models (GCMs) must invoke substantial additional approximations since climate evolution is dictated mainly by the atmosphere, and the oceans follow inertial responses that significantly exceed those of the atmosphere. Estimation requirements therein are therefore not compelling. Climate projections based solely on GCM outputs suffer from uncertainty related chiefly to computational cost when generating the required ensemble of long-running simulations. The development of multi-scale modeling frameworks able to provide climate-scale runs at acceptable expenses therefore remains of utmost priority (Beucler et al., 2021)

**General Circulation Models and Energy Balance Models**

A General Circulation Model (GCM) simulates the coupled atmosphere, ocean, land, and ice components of the Earth's climate system, allowing for projections of future climate changes from prescribed greenhouse gas emissions (J. Valdes et al., 2017). Energy Balance Models (EBMs) capture the equilibrium climate that arises from the balance between incoming solar radiation and outgoing terrestrial radiation, providing a physically based, global approximation of the climate response and accompanying feedbacks that is tractable even though the governing equations are nonlinear (W. Robertson and Ghil, 2000). Both approaches provide valuable tools with different but complementary insights into climate dynamics. GCMs deliver detailed projections of flows, energy, and mass transport over time, while EBMs characterize fundamental climate principles and highlight prominent feedback mechanisms.

**Multi-Scale Modeling and Embedding Approaches**

Multi-scale modeling and embedding approaches can serve as a starting point for the development of multi-scale models to provide relevant information and guidance for impacts assessment. The climate system is inherently multi-scale, exhibiting phenomena across an extensive spectrum of spatial and temporal scales. These scales are often interlinked. For example, tidal motions and storm surge associated with astronomical tides—interacting with wind-induced sea-level pressure. A key issue is how to model processes—such as crop growth—that span a range of scales within a multi-scale modeling framework. Traditional computer models often retain a significant degree of detailed formulation, with some sub-models operating without fully resolved boundary conditions, leading to confusion regarding models—particularly those that make use of spectral embedding approaches—that lack the high-resolution simulation data in which scale interactions are represented. A latent-space multi-scale framework has been investigated to address the complex trade-offs between model complexity and structure both within and across scales (Singh et al., 2024). Several facilities are available within the Climate Office in the ICRC, including a suite of Earth System Models.

**Emulation, Surrogate Models, and Emulator Validation**

The proliferation of complex Earth System Models (ESMs) has reignited interest in climate modeling and emulation. This has led to the characterization of the MAGICC Temperature Emulator clime model, implemented within a reduced-complexity framework, for efficient screening of climate and impact assessments. A surrogate-level machine-learning-based endeavor, dubbed the MAGICC Emission Temperature Emulator, aims to comprehend the interrelations between emissions and temperature up to a duration of 2050–2100 under several scenarios. It leverages a LIGHTGBM-based regressor for sophisticated emulation within planetary atmospheres subjected to anthropogenic influences while being trained exclusively upon emission–temperature pairs. The assessment reveals that tackling the generation of uncorrelated scenario sets is instrumental for the strategy, as conventional characteristics deriving from datasets such as the Representative Concentration Pathways (RCPs) constitute high-dimensional aspects closer to the domain of MAGICC emissions, and certain linkages are more readily comprehended when purely emission-dependent approximations are contrived (Miftakhova et al., 2020). Further, the increasingly prohibitive computational burden of high-dimensional Earth System Models (ESMs) and the need for computationally affordable high-dimensional surrogates for a wide range of emissions scenarios have underscored the importance of successful emulation (Bouabid et al., 2023).

Uncertainty Quantification in Climate Forecasts

Considerable uncertainties are associated with climate predictions from numerical models. Betts (2020) categorized present-day uncertainties into four classes distinguished by the nature of their characterization: parametric, structural, initial-condition, and forcing—that is, the construction of the equations, external constraints or laws of motion, and the physical inputs to the system, respectively. Additional sources of uncertainty, commonly grouped under the broader heading of model-form uncertainty, are related to the representation of certain processes it is found appropriate to incorporate (Strobach and Bel, 2017). Lucarini (2004) considered the classification of climate modeling uncertainty on a more aggregated level. These important guidance frameworks motivate the development of techniques for uncertainty quantification and are used to structure the exposition below.

Statistical methods for uncertainty propagation are increasingly being integrated into forecasts and rely broadly on concepts and techniques from the theory of random dynamical systems. The discussions encompass both the basic concepts and the main classes of models employed. The methodology aligned with the conventional one-parameter-at-a-time approach of sensitivity analysis, which studies the behavior of the model at specific parameter values, is termed the "flyovers" technique.

More advanced yet still relatively low-cost variants founded on polynomial chaos or on the combination of bootstrapping with ordinary differential equations are outlined. On the climate-prediction side, applicability to the entire ensemble rather than just to individual members is recognized as a crucial attribute. Those approaches closely related to conditional probabilities and capable of yielding probabilistic forecasts are briefly identified. Techniques developed under the umbrella of sensitivity analysis are examined.

**Sources and Characterization of Uncertainty**

Climate projections and scenarios provide valuable information across a wide range of time scales and are essential for understanding possible climate development under alternative greenhouse gas emission pathways. Despite a substantive knowledge of the climate system, making predictions of future climate remains challenging. The atmospheric and oceanic components of the climate system vary on different time scales depending on a multitude of processes, ranging from the seasonal cycle to the precession cycle or slow change of the orbital parameters. Consequently, a single prediction horizon cannot provide enough information about future evolution in the climate. Furthermore, the estimates of future climate situations vary according to the emission scenario used and therefore the probability associated with any climate projection is a key issue for society to assess the risk. A single curve, or its consistently paired intervals, is insufficient to convey this information.

Uncertainty quantification has become a key topic in the climate modelling community. The IPCC defines five sources of uncertainty surrounding future climate predictions: scenario uncertainty, emission uncertainty, parameter or structural uncertainty, natural variability uncertainty, feedback uncertainty . The above issues can be grouped into different categories: structural uncertainty is linked to the model-form uncertainty and after consequent developments in the climate model intercomparison project is well constrained. The feedback uncertainty is associated with the climate sensitivity evaluation; natural variability is several orders smaller than the expected signal after a few decades, and climate model has an increasing competence in those aspects. The remaining issue is related to the emission uncertainty as the coupling between economy development and GHG emissions or the representation of the economic model in climate-economy modelling framework needs further robust investigation. However, the corresponding uncertainty quantification for carbon cycle and hence model structural uncertainty which is tightly bound with the emissions remains largely unexplored.

**Statistical Methods for Uncertainty Propagation**

Dynamical systems provide a natural framework for describing large-scale climate dynamics. Governing equations are often expressed in terms of partial differential equations that may represent nonlinear interactions among geophysical flows on different time and space scales. In theory, such deterministic climate models define dynamical systems for which basic concepts from the theory of dynamical systems—including equilibria, stability, bifurcations, and invariant sets (e.g., attractors)—are applicable (A. Thomas and Lin, 2015).

Large-scale climate dynamics exhibit high-dimensional yet structured trajectories whose persistence through long timescales motivates the use of reduced models. Generating low-order representations that satisfactorily reproduce relevant aspects of climate evolution and variability, while maintaining consistency with the underlying high-dimensional model, has dragged the attention of the research community over many decades. Despite evident progress, deriving such low-order representations remains challenging (Strobach and Bel, 2017).

**Probabilistic Forecasts and Reliability Assessment**

Probabilistic forecasts provide both a numerical prediction of states and an estimate of the associated uncertainty. Observed probabilities should match forecast probabilities over ample verification. Hence, calibration relies not only on reliability diagrams but also on the use of proper scoring rules that take into account the probabilistic nature of the forecasts. Multi-step-ahead density forecasts call for a specific treatment of calibration targets (Aizenman et al., 2016). The evaluation of forecast quality includes verification frameworks to characterize the predictive value of the model, addressing not only the physics of climate but also the reliability of model-simulated climate variability and trends (J. van Oldenborgh et al., 2013).

The course of climate interventions often concerns regional extreme events like droughts or quality alerts. Regional climate models may then be employed to assess these forecasts. When running ensemble forecasting systems, the introduction of independent stochastic noise at the model level becomes necessary to avoid undesired correlation in the first-guess forecasts (Lerch, 2016). It is also advisable to check consistency of the stochastic forcing across model components. Although initial or boundary conditions can still be conditioned upon forecast models involving large-scale features, proper-spreading noise can be introduced over the remaining degrees of freedom.

Model Evaluation and Validation

Models need to be highly reliable at prediction in order to form a valid basis for decision making. Model evaluation and validation procedures help to ascertain this reliability. Hindcasting facilitates the analysis of prediction capacity at earlier dates, in a manner identical to present-day forecasts, or by using still earlier model configurations. Cross-validation involves withholding a fraction of measurements from adjustment procedures, whether performed through calibration or tuning, and applying the resulting model to predict the omitted data. A variety of skill metrics are available, both for verifying specific forecast attributes and for evaluating overall predictive quality (Fildes and Kourentzes, 2011). Benchmark simulations, whether for atmospheric dynamics, climate sensitivity or other attributes, can also gauge absolute model performance and compare it against that of established alternatives. The degree of skill relative to a reference model can be quantified via skill scores.

Climatic models' transparency promotes critical appraisal of structural validity and observational consistency. Ability to produce a specific historical signal—hence support for the hypothesis that the same signal emerges under unobserved boundary conditions simultaneous with those that were controlled or monitored—remains an important validation criterion. Many models nonetheless achieve substantial skill in real-time predictions conditioned on such assimilation residue. Data obtained from assimilation contribute similarly to benchmarking. Infrastructure for continually augmenting observational data exists, but quality assurance, doubt resolution and adjustment for non-statistical influences warrant attention (L Kent, 2018). Various homogenization situations arise when considering both reference and targeted elements, hampering attempts to define a truth estimate, while establishment of such a truth estimate remains a prerequisite for rigorous redundancy checking.

**Hindcasting, Cross-Validation, and Skill Metrics**

Hindcasting is a pivotal tool for testing model assumptions and subsequently providing the foundation for meaningful climate predictions. It mitigates the risks associated with committing large resources to scenarios based exclusively on climate models designed for future time. Cross-validation is a robust generalization of hindcasting. It enables the modeling of aspects of the climate system that cannot be easily obtained from short observational records or, alternatively, that have undergone substantial changes since the start of those records. Climate variability can be detected at hidden temporal scales much longer than those represented in observations (Fildes and Kourentzes, 2011). The calibration of realistic climate models is far from achieving confidence, and the exploration of alternative models of the evolution of temperature or other variables, such as CO2 concentration, remains vital. Skill metrics are crucial for distinguishing models producing a suppression or increase of those variables. They provide a rich range of choices for assessing the relevance of various climate aspects either in the framework of the same working model or with significant changes in the climate paradigm, thus evaluating the predicted warming expansion or two-fold increase of CO2 concentration and the prospects for a new ice age if other models are employed (Aizenman et al., 2016).

Since one of the first model evaluations in climate science, a distinction has been made between the fundamental goals of modeling and formal comparisons of simulations to conveniently processed or filtered observations that may not be addressed by the model systems under study. The adequacy of the actual simulated dynamics can be determined through a range of model-specific and observationally obtained diagnostics of dynamical regimes. Particular attention has been devoted to methods of assessing climate model simulations of atmospheric circulation owing to the crucial role of the atmosphere in climate change.

**Observational Data Assimilation and Benchmarking**

Climate change agendas advocate the introduction of sound climate policies and combat global warming, driving society to progress toward net-zero emission strategies through technological innovation and human ingenuity.

The success of these objectives will depend on the future trajectories of fc1e7a7f-06c9-472e-896c-73b24203a2ffhouse gas emissions and the response of climate systems to those emissions. Accordingly, Governments' climate policy decisions are built upon the scientific community's ability to predict future climate change and its impacts.

Two purely mathematical modelling classes, energy balance (EBM) and impulse response (IRM), are commonly identified as benchmark climate-models. They are employed to reduce the climate-form modelling burden and satisfy policy-analyses demands. These models are derived from efficient, lower-, and larger-time-scale reduction, diffusing climate change from a computationally expensive, complicated climate code using a single state variable or climate component. EBM focuses on the atmospheric-temperature representation while IRM works from emission input to temperature response. As the EBMs' simplicity attracted the intellectual community forty years ago (C Green et al., 2008) , the temperature elements coupled to an impulsive-with-histories forcing remain a useful formalism.

Policy-Relevant Climate Predictions

Climate models provide climate projections that help quantify potential changes and associated risks due to global warming. Projections in general circulation models (GCMs) are often detailed but, given their complexity, disconnected from climate-related decisions. Consequently, several integrated assessment models (IAMs) drive policy agendas employing reduced-form energy balance or energy-economic models focused on carbon dioxide stabilization targets and socioeconomic trajectories. Multi-scale frameworks embedding GCMs in IAMs address the necessity of connecting climate science to policy-relevant decisions concerning mitigation and adaptation strategies. These computable-prototype or infrastructure-observatory models vary in complexity and sophistication from IAMs down to simple energy balance formulations. A variety of climate prediction models exist, providing projections in line with contemporary climate science – projections compatible with two different climate models, thus reflecting relevant science-policy linkages. A continuum of climate-prediction frameworks ranges from complex GCMs to simple internationally compatible regulatory-level assessments and national-level policy-driven energy-economic analyses (J. Challinor et al., 2013).

**Scenario Analysis and Risk Framing**

The Intergovernmental Panel on Climate Change (IPCC) Special Report on Global Warming of 1.5 °C (IPCC, 2018), consolidating the results obtained by three groups of integrated assessment models (IAMs), stressed the importance of scenario analysis in guiding the ongoing climate debate between technological optimism and pessimism. Representative concentration pathways (RCPs) are commonly used as scenario indicators. These scenarios describe alternative stories of the future to assess climate change impacts that differ into the mid-21st century (Tomassini et al., 2010). Both greenhouse gas emissions and associated socioeconomic drivers, global and by region, determine the RCPs. Risks related to climate change are already affecting several sectors and are expected to increase. Mitigation policies can significantly reduce associated climate-related risks (A. Winkler et al., 2010). Adaptation deals with societal preparedness to cope with ongoing and anticipated climate changes and enhances the resilience of human and natural systems. In addition to the RCPs, an alternative framework for long-term decision-making under deep uncertainty (DMDU) was developed to address the selection of adaptation strategies along a ten-year planning horizon focusing on the decision-relevant extreme rainfall (https://ccafar.org/climate-complexity-access-toolkit/2-1-uncertainty-resources/a-brief-introduction-to-dmd/).

**Decision-Focused Forecasts and Extreme Event Prediction**

Institutional and societal decisions increasingly depend on climate information (Lerch, 2016). Tailoring climate models to meet decision-focused needs enhances predictions of extreme events (Fildes and Kourentzes, 2011). Probabilistic, model-based approaches facilitate the consideration of changes to climate parameters, and situations where calibration cannot be justified remain an active area of research.

In decision-focused climate modeling, the goal is to provide information relevant to specific decisions. Decision-framing approaches specify the decision context and its interconnections with climate parameters. Based on this framing, stages of the decision process can be framed in terms of risk, and representative concentration pathways (RCPs) formulated to reflect the range of trajectories associated with common climatic shifts.

Decision-focused climate projection emphasizes the consideration of two further stages, which concern the anticipated influence of climate fluctuations on the underlying socioeconomic condition and on the specific decision of interest. These components enable more granular consideration of climate influences from a firm and an economy-wide perspective.

Challenges, Limitations, and Open Research Questions

Mathematical modeling is central to climate science, which aims to understand the Earth's climate, determine anthropogenic influences, and provide projections. Such models are typically either empirically based, prescribed at the commencement and continuously updated, or they encompass governing equations, initial conditions, and external forcing (Lucarini, 2004). The mathematical investigation of climate focuses on the latter type. Climate models embody dynamical systems, and the study of their generic properties yields insight into climate phenomena.

The future of climate forecasting is contingent upon resolving open questions. Model output suggests responses of the Earth's climate system to the deployment of greenhouse gases may lie at or beyond the upper limits of the probabilistic range presently considered (Fildes and Kourentzes, 2011). Yet governing equations for the atmosphere and oceans permit robust climate forecasting on a time horizon of up to two decades.

Projections consequent to climate policy scenarios exhibit historical validation problems, constraining confidence. Uncertainty quantification for climate predictions is fundamentally problematic: too many climate models for the present climate situation yield reliable hindcasts, and decadal prediction skill remains inadequately understood. Because of their physical basis, climate models retain structural uncertainty. Such models may be operatively distinguished from weather, weather-forecasting, and weather-forecasting models.

Conclusion

The issue of climate change is significant in such a manner that if it remains unanswered, modern civilization is in peril. Mathematics plays a major role in understanding it and decides courses of action to increase sustainability and mitigate climate change effects in the process of formulating and improving models. Climate science therefore benefits from an active engagement with the mathematical sciences. Addressing climate change at all time scales requires the ability to model the time-varying state of the Earth's climate, and models based on the laws of physics can inform projection of future behaviours of climatic systems. Climate modelling relies strongly on mathematics and many subjects from pure mathematics to applied mathematics and statistics are involved (Fildes and Kourentzes, 2011).

Prediction is critical in understanding climate processes and is needed for monitoring, extreme-event assessment and risk determination, space missions, and other high-impact applications. A broad range of climate phenomena spans time scales from minutes to millennia, but the mathematical approaches used to study and model them share important commonalities across different time ranges. Depending on the range of interest, approaches to deal with complexity diverge significantly, necessitating models that operate at drastically different physical and time scales. Observed climate time series permits modelling of the emergence of large-scale truths from a throng of short-term fluctuations. Moreover, established climate scenarios form a vital science-policy interface linking predictions of climate models to decisions on mitigation and adaptation strategies with societal implications.

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